Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department

Final Term Exam Date:15-1-2015
Mathematics 3A Code : EMP281
Duration : 3 hours

- Answer all the following questions
- The exam. Consists of **one** page
- No. of questions: **8**
- Total marks: **80** [**10 marks each**]

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1) Susan goes to work by one of two routes A or B. The prob. of going by route A is 30%. If she goes by route A, the prob. of being late is 5% and if she goes by route B, the prob. of being late is 10%. Given Susan is late for school, find prob. that she went via route B.

2) A fair coin is tossed three times, X is the N^o of heads that come up on the first 2 tosses and Y is the N^o of heads that come up on tosses 2, 3. Construct the joint distribution and find marginal of X and Y, also find expected value and variance of X , Y & P [(X + Y) >1, X > Y].

3) A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of (a) More than 2 hits? (b) At least 3 miss?

4) A particular county in Louisiana experienced incidents of West Niles virus at an average rate of **2.6** per month. What is the probability of at least three persons coming down with West Niles virus during a month?

5) If $f(X) = c\,x^2 e^{-2x}$ is P.d.f., $x > 0$, find the mean and variance and M.G.f.

6) Expand in Fourier series the following functions:

i) $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ -(x-\pi), & \pi < x \le 2\pi \end{cases}$, ii) $f(x) = x^2$, $0 < x < 2$.

7) Suppose a shipment of 100 VCRs is known to have 10 defective VCRs. An inspector chooses 12 for inspection. He is interested in determining the probability that, among the 12, at most 2 are defective.

8) Expand in Fourier series the following periodic function $f(x) = |\sin x|$ $0 < x < 2\pi$, then deduce

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} - .... \quad \text{and} \quad \frac{1}{1^2.3^2} + \frac{1}{3^2.5^2} + \frac{1}{5^2.7^2} + ...$$

Questions	Total marks	Achieved ILOS	Questions	Total marks	Achieved ILOS
Q1	10	b1	Q5	10	b7
Q2	10	a1	Q6	10	c1
Q3	10	a5, c1	Q7	10	a1, b1
Q4	10	b2	Q8	10	c1

Board of examiners: Dr. eng. Khaled El Naggar

Model answer

1) P(A) = 0.3, P(B) = 0.7, P(L/A) = 0.05, P(L/B) = 0.1, P(L) = 0.05(0.3) + 0.1(0.7) = 0.085, P(B/L) = 0.1(0.7)/0.085= 0.8235

2)

Y \ X	0	1	2	$f_1(x)$
0	1/8	1/8	0	2/8
1	1/8	2/8	1/8	4/8
2	0	1/8	1/8	2/8
$f_2(y)$	2/8	4/8	2/8	1

E(X) = 0(2/8) + 1(4/8) + 2(2/8) = 1, E(Y) = 0(2/8) + 1(4/8) + 2(2/8) = 1, $E(X^2)$ = 1(4/8) + 4(2/8) = 3/2 , $E(Y^2)$ =1(4/8) + 4(2/8) = 3/2, Var(X) = Var(Y) = 1/2.

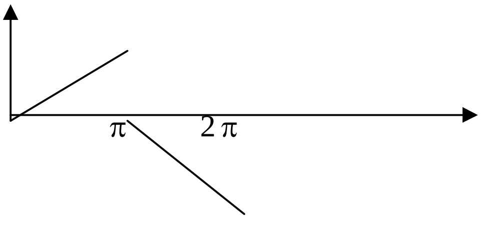
P [(X + Y) >1, X >Y]= P(2,0) + P(2,1) = 1/8

3) This problem can be solved by binomial distribution and let N^0 of hit is the random variable such that p = 0.8, q = 0.2, therefore $P(X > 2) = \sum_{x=3}^{4} {}^4c_x (4/5)^x (1/5)^{4-x}$ and at least 3 misses equal at most one hit the target = $P(X \le 1) = \sum_{x=0}^{1} {}^4c_x (4/5)^x (1/5)^{4-x}$

4) $P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, $\lambda = 2.6$, thus $P(X \ge 3) = 1 - P(X \le 2) = 1 - \sum_{x=0}^{2} \dfrac{e^{-2.6}(2.6)^x}{x!}$ =0.518

5) Since $f(X) = 4\, x^2 e^{-2x}$ is gamma distribution with $\alpha = 3$ & $\beta = 2$, therefore $E(X) = \alpha / \beta = 3/2$ and variance is $\alpha / \beta^2 = 3/4$ and M.G.f = $\dfrac{\beta^\alpha}{(\beta - t)^\alpha} = \dfrac{2^3}{(2-t)^3}$

6) i) The function is odd harmonic, therefore



$$a_{2n-1} = \frac{2}{\pi}\int_0^{\pi} x\cos(2n-1)x\,dx = \frac{2}{\pi}\left(x\left(\frac{\sin(2n-1)x}{2n-1}\right) - \left(\frac{-\cos(2n-1)x}{(2n-1)^2}\right)\right)\Big|_0^{\pi} = -\frac{4}{\pi(2n-1)^2}$$

$$b_{2n-1} = \frac{2}{\pi}\int_0^{\pi} x\ \sin(2n-1)x\,dx = \frac{2}{\pi}\left(x\left(\frac{-\cos(2n-1)x}{2n-1}\right) - \left(\frac{-\sin(2n-1)x}{(2n-1)^2}\right)\right)\Big|_0^{\pi} = \frac{2}{(2n-1)}$$

Thus $f(x) = \sum_{n=1}^{\infty} a_{2n-1}\cos(2n-1)x + \sum_{n=1}^{\infty} b_{2n-1}\sin(2n-1)x$

ii) This function is $f(x) = x^2$ neither even nor odd, therefore

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{T}\right) + b_n\sin\left(\frac{n\pi x}{T}\right) \quad , T=1$$

$$a_0 = \frac{1}{1}\int_0^2 f(x)\,dx == \frac{1}{1}\int_0^2 (x^2)dx = \left(\frac{x^3}{3}\right)\Big|_0^2 = \frac{8}{3}$$

$$a_n = \frac{1}{1}\int_0^2 (x^2)\cos\left(\frac{n\pi x}{1}\right)dx = \left[(x^2)\left(\frac{1}{n\pi}\sin\left(\frac{n\pi x}{1}\right)\right) - (2x)\left(-\frac{1}{n^2\pi^2}\cos\left(\frac{n\pi x}{1}\right)\right)\right.$$

$$\left. + \ 2\left(-\frac{1}{n^3\pi^3}\sin\left(\frac{n\pi x}{1}\right)\right)\right]\Big|_0^2 = \frac{4}{n^2\pi^2}$$

$$b_n = \frac{1}{1}\int_0^2 (x^2)\sin\left(\frac{n\pi x}{1}\right)dx = \left[(x^2)\left(\frac{-1}{n\pi}\cos\left(\frac{n\pi x}{1}\right)\right) - (2x)\left(-\frac{1}{n^2\pi^2}\sin\left(\frac{n\pi x}{1}\right)\right)\right.$$

$$\left. + 2\left(\frac{1}{n^3\pi^3}\cos\left(\frac{n\pi x}{1}\right)\right)\right]\Big|_0^2 = \frac{-4}{n\pi}$$

7) n= 12, N = 100, k= 10, therefore $P(x \le 2) = \sum_{x=0}^{2}\left[{}^{10}C_x\right]\left[{}^{90}C_{12-x}\right] / \left[{}^{100}C_{12}\right]$

8) This function is even cosine harmonic, therefore $a_0 = \frac{4}{T}\int_0^{T/2} f(x)\,dx = \frac{4}{\pi}\int_0^{\pi/2}\sin(x)\,dx = \frac{4}{\pi}$

$$a_{2n} = \frac{4}{T}\int_0^{T/2} f(x)\cos\left(\frac{2n\pi x}{T}\right)dx = \frac{4}{\pi}\int_0^{\pi/2}\sin(x)\cos(2nx)\,dx = -\frac{2}{\pi(2n-1)(2n+1)}\ ,\ b_{2n} = 0$$

Thus $|\sin x| = \frac{a_0}{2}\sum_{n=1}^{\infty} a_{2n}\cos(2nx) = \frac{2}{\pi} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}\cos(2nx)$, put x = 0, therefore $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)} = 1$

By Parseval's theorem, $\frac{2}{T}\int_0^T [f(x)]^2 dx = \frac{a_0{}^2}{2} + \sum_{n=1}^{\infty} a_{2n}^2 = \frac{1}{\pi}\int_0^{\pi}(1-\cos 2x)dx = 1$,

Therefore $1 = \frac{8}{\pi^2} + \sum_{n=1}^{\infty}\frac{16}{\pi^2(2n-1)^2(2n+1)^2} \Rightarrow \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2(2n+1)^2} = [1 - \frac{8}{\pi^2}]\frac{\pi^2}{16}$